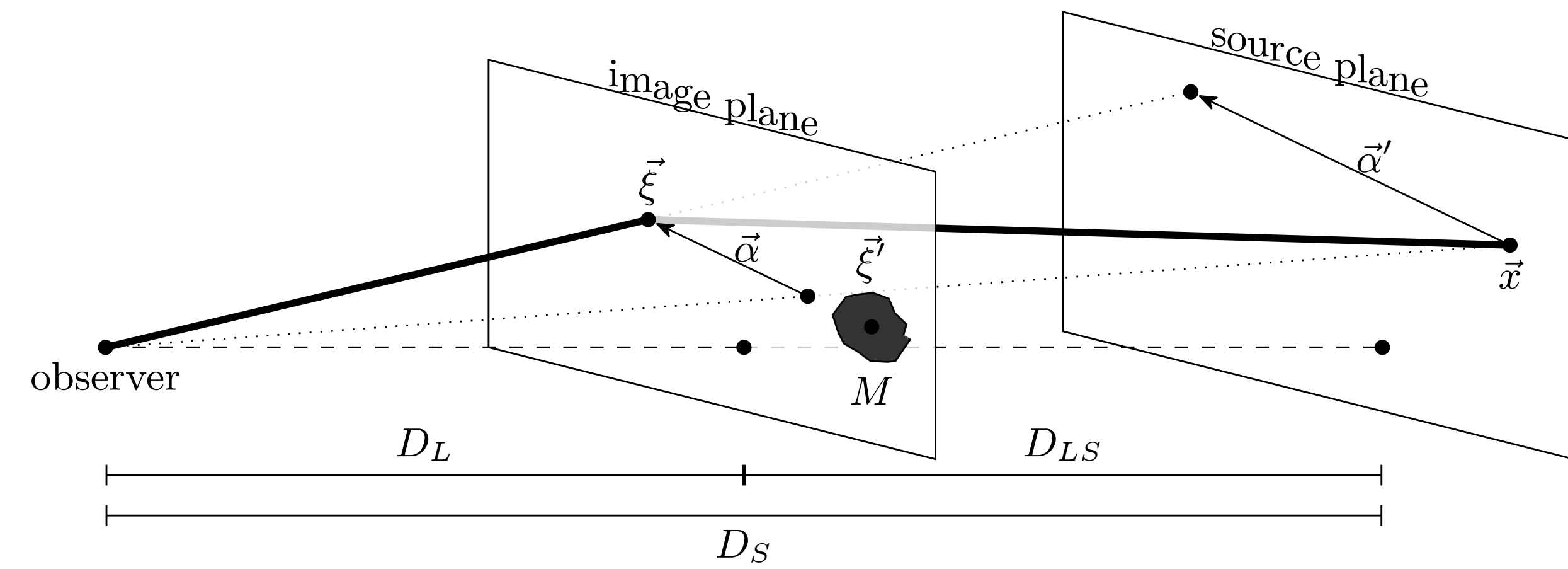


Lensing and dark matter substructure

Strong gravitational lensing uniquely probes **dark matter subhalos**



Goal: compute posteriors for subhalo position and mass

Difficulties: complex correlations between image pixels; marginalization over many model parameters & subhalos

Methodology:

1. Fit **approximate posterior** for source and lens model parameters to an observation. Posterior acts as a **targeted simulator** to producing data similar to the observation.
2. Train **likelihood-free inference** network to compute substructure posteriors, marginalizing out model parameters

A new model for gravitational lenses

Standard GP

$$\mathbf{f} \sim N(0, \mathbf{K})$$

$$\mathbf{x} \sim N(\mathbf{f}, \sigma_n^2)$$

$$\mathbf{K} = \alpha^2 \mathbf{T} \mathbf{T}^\top$$

$\mathbf{f}$: true fluxes in each pixel

$\mathbf{x}$: observation

σ_n : observation noise

Our model

$$\mathbf{y} \sim N(0, \alpha^2)$$

$$\mathbf{f} = T(\mathbf{p}, \sigma) \mathbf{y}$$

$$\mathbf{x} \sim N(\mathbf{f}, \sigma_n^2)$$

$\mathbf{y}$: source parameters (1/pixel)

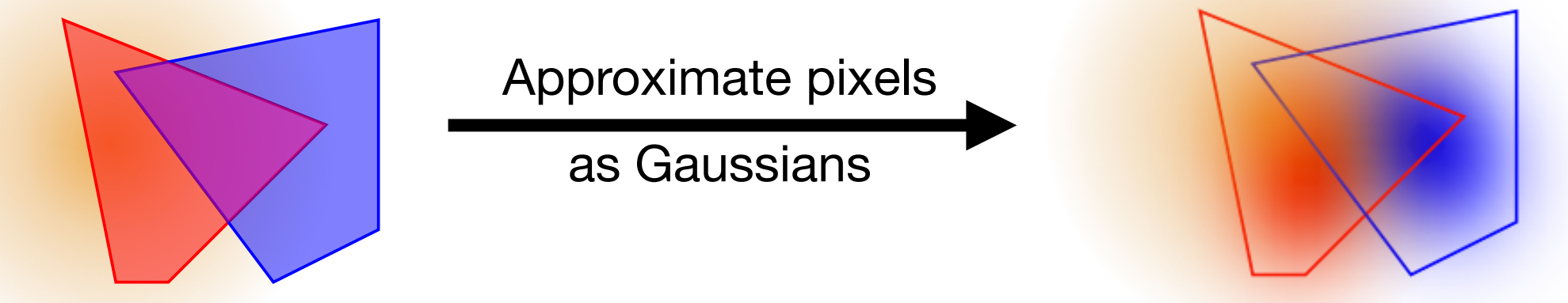
α : variance hyperparameter

σ : kernel size hyperparameter

$\mathbf{p}$: pixel coordinates

Covariance $\mathbf{K}$ induced by **intrinsic source variations** & **pixel overlaps** in source plane:

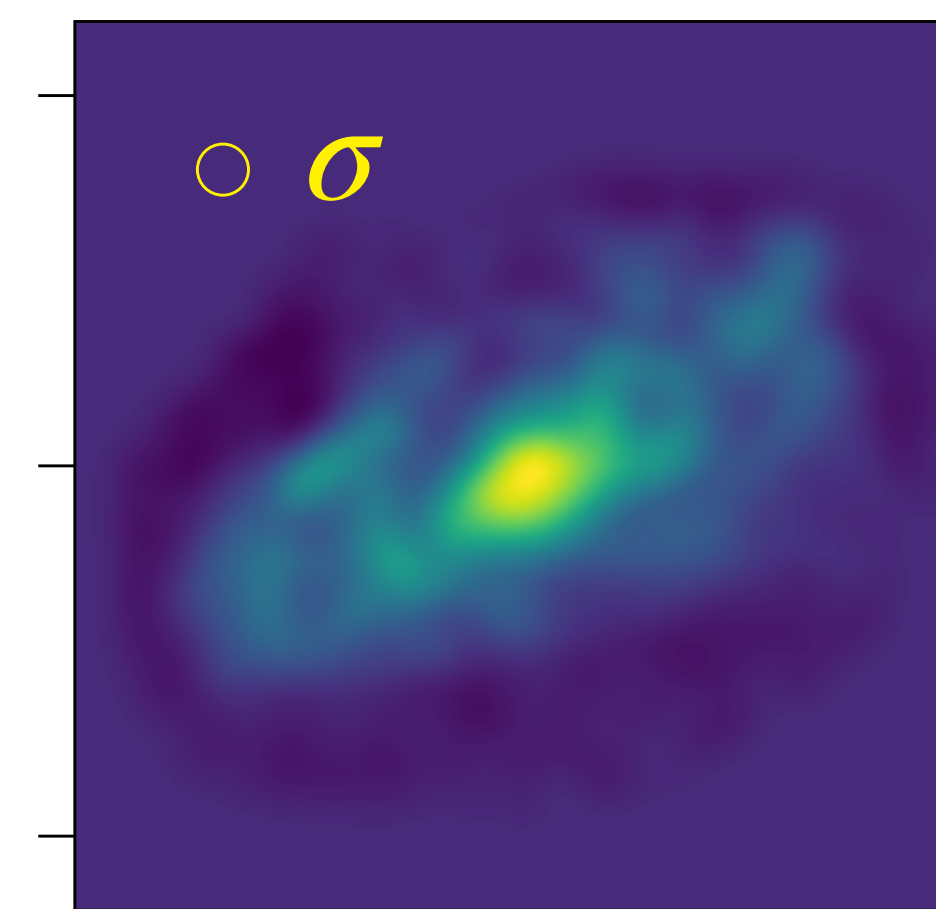
$$\mathbf{K}_{ij} = \iint d\mathbf{p}_1 d\mathbf{p}_2 g_i(\mathbf{p}_1) k(\mathbf{p}_1, \mathbf{p}_2) g_j(\mathbf{p}_2)$$



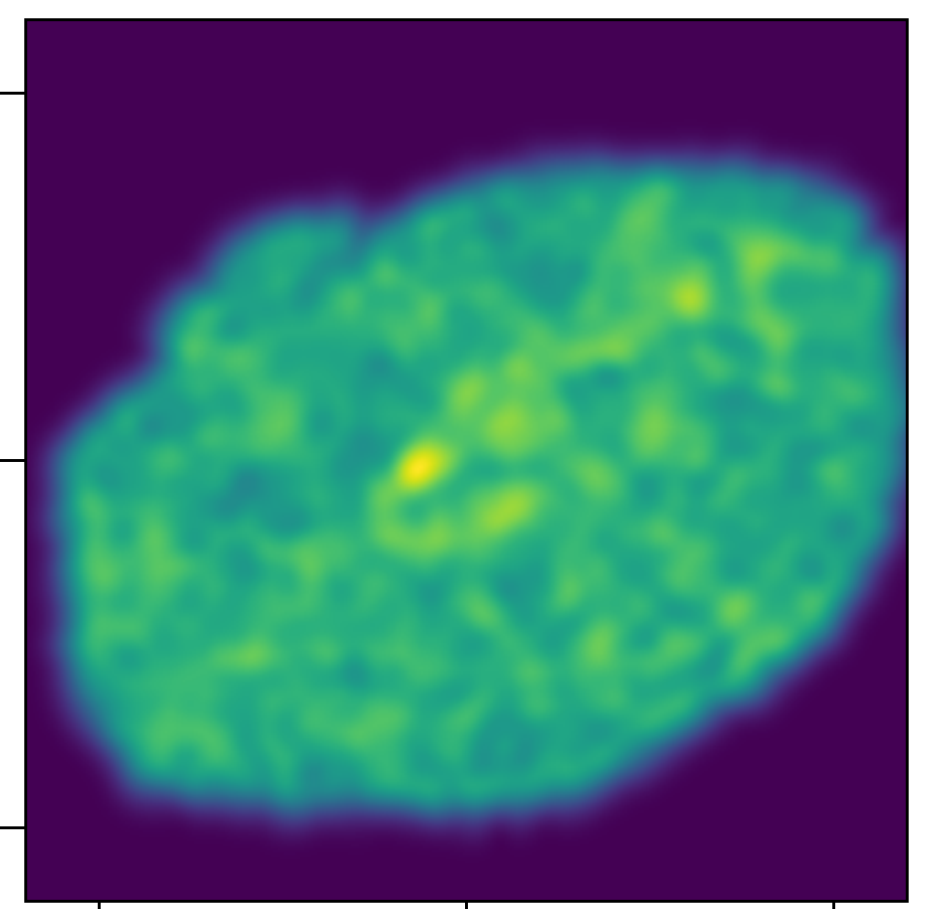
Can approximate $\mathbf{T}$ using fact that matrix multiplication $\sim$ spatial convolution

- Instead of infeasible GP matrix inversion, fit **variational posterior** for lens & source parameters to observation, *ignoring substructure*
- **ELBO maximization** via gradient descent with reparametrization trick (using pyro)
- ➔ Requires lens model to be automatically-differentiable (implemented in pytorch)
- Simultaneously optimize α hyperparameter with σ fixed by hand
- We use a diagonal-normal posterior for the $\sim 10^5$ source parameters & multivariate normal for lens parameters

Source mean



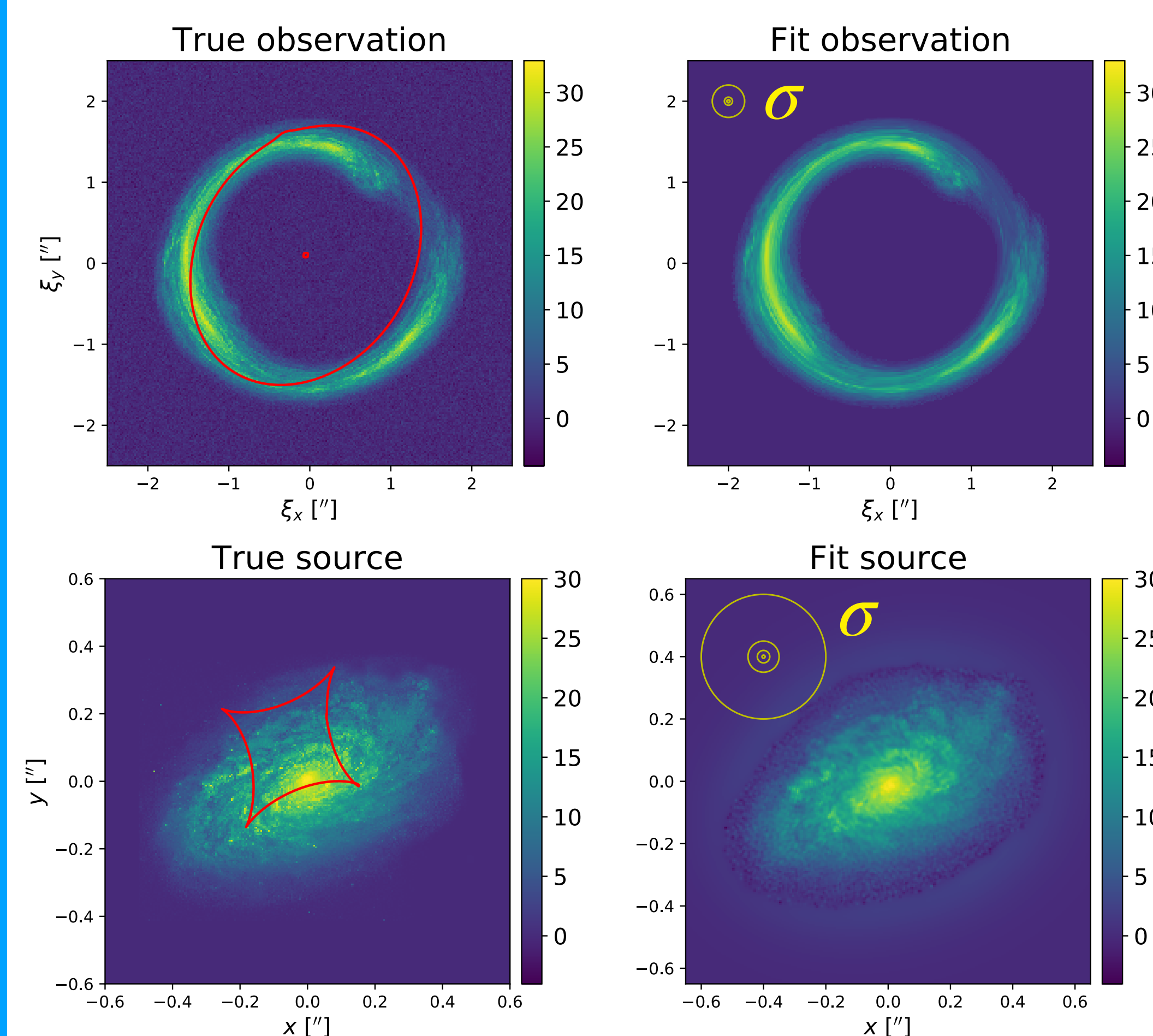
Source uncertainty



Substructure inference

- **Aim:** posteriors for substructure parameters $\mathbf{z}_d$ marginalized over source and lens parameters
- **Neural likelihood-to-evidence ratio estimation:** estimate $p(\mathbf{z}_d | \mathbf{x}) / p(\mathbf{z}_d)$
- Train classifier $d(\mathbf{x}, \mathbf{z}_d)$ to distinguish two classes:
 1. $\mathbf{x}, \mathbf{z}_d \sim p(\mathbf{x}, \mathbf{z}_d)$ data & parameters sampled from joint distribution
 2. $\mathbf{x}, \mathbf{z}_d \sim p(\mathbf{x}) p(\mathbf{z}_d)$ sampled independently
- ➔ Can recover ratio through $d(\mathbf{x}, \mathbf{z}_d) / (1 - d(\mathbf{x}, \mathbf{z}_d))$
- **Training samples:** draw from approximate posterior
- Implemented using swyft (see [paper](#) & poster at this workshop!)
- Mock data analysis of high-resolution image: only 10,000 training samples, runs in a few hours on single GPU. Promising results!
- **Next up:** apply to real Hubble Space Telescope data

Mock analysis: fit results



Mock analysis: subhalo inference results

